MATHEMATICAL AWARENESS

Preamble

The above-mentioned course on Mathematical Awareness is designed to create appreciation of the beauty and importance of mathematics in our everyday lives. Care has been taken to gently yet purposefully introduce the student to some central yet elementary aspects of mathematics. The contents are to be taught with emphasis on the use of intuition, and history rather than on rigour. Skills for visual interpretation shall be brought into focus and yet some theoretical aspects with stress on enhancing numerical ability shall be developed.

Mathematical Awareness

The aim of this course is to enhance mathematical ability and increase awareness of mathematical developments, with a special relevance to the awareness of mathematical development, with a special relevance to the real world. Keeping these objectives in view.

* the emphasis will be on an intuitive approach
* historical background relevant to each topic will be given and
* special emphasis will be on skill of visual interpretation and enhancing numerical ability

The syllabus for this course is given below

Unit I - Brief Biographical Sketches **6 marks**

A brief introduction to the lives and information on the works of the following Mathematicians:

Archimedes, Euclid, Pythagoras, Aryabhata, Bhaskaracharya II, Brahmagupta, Madhavacharya, Neelkantha, Newton, Cauchy, Euler, Abel, Galois, Gauss, Germain, Kovalyskaya, Noether, Riemann, Hilbert, Noether and Ramanujan, Harish Chandra.

Of the above, biographies of Euclid, Newton, Noether, Ramanujan and Riemann will be examined.

The biographies of the remaining mathematician in the above list are to be covered via projects for internal assessment.

Unit II - Numbers **12 marks**

An overview of number systems, including algebraic and transcendental numbers, with some historical background.

Divisibility of integers, The Euclidean algorithm, Modular Arithmetic and some divisibility criteria. Magic squares.

Prime numbers, the sieve of Erastosthenes, the fundamental theorem of arithmetic, Euclid's Theorem. Mersenne Numbers and Mersenne Primes, Goldbach Conjectures, Prime testing method of Fermat, Statement and significance of The Prime number theorem. Uses of prime numbers, for example in RSA.

Pythagorean triples, Statement and historic background of Fermat's Last Theorem. Fibonacci sequences in nature.

Multiplication Principle, Permutation and Combinations.

The emphasis will be on enhancing numerical ability through these concepts.

Unit III – Graph Theory and Geometry **12 marks**

Basics of Graph theory, the Konigsberg Bridge problem, Four-Colour map theorem.

Introduction of functions, graphs of functions. Increasing and decreasing functions, even and odd functions, location of points of extrema, inflection, periodic functions-all via graphs.

Perspective and Projection. Perspective geometry: lines and points in 2D and 3D, Fundamental trigonometric functions. Use of perspective in drawing, historical background. Common tools adopted by artists for such representations. Analysis of some paintings to spot use of perspective and techniques. Escher's art.

Types of symmetry, concrete examples of symmetry groups, Basic tilings.

Study of symmetry and patterns by looking at monuments/buildings/ornamental art.

Shapes and solids. The regular Polyhedra, Euler's formula, Golden ratio, Golden rectangle.

The importance of Platonic solids and mystical significance to the ancient Greeks. Construction of Altars and geometry in ancient India.

Fractals in nature, for example snowflakes and coastlines.

The Mobius Strip and the Klein Bottle.

The emphasis while dealing with the above topics will be on enhancing skills of visual perception and interpretation.

Unit IV – Statistical and Related Concepts **8 marks**

Histograms, pie charts, tabular forms and ogives. Mean, Median, Mode, Variance, and Standard Deviation. Data analysis, interpretation and prediction.

Classical approach to probability, the addition and multiplication laws of probability.

Solutions of linear inequalities using graphs. Historical development of operations research.

Use of computer based spreadsheets for simple statistical analysis.

Project work for internal assessment **12 marks**

Projects based on one of the following:

1. Data analysis, interpretation and prediction using an appropriate computing tool.
2. The biographies of the mathematicians listed above. (Excluding the five that are examinable)

Texts

* Gulberg, Jan*, Mathematics: from the birth of numbers,* W.W. Norton and Company, New York & London, 1996.

* loan, James, *Remarkable Mathematicians,* Combridge University Press, 2004.

* Salkind, Neil J., *Statistics for people who (think they) hate Statistics,* Sage Publications, Inc. 2000.

Supplements

* Bibhutibhusan Datta, A*ncient Hindu Geometry : The Science of the Sulbas,* Calcutta Univ. Press, 1932, Reprinted, Cosmo. Pub., New Delhi, 1993.

* Farmer, David, W., *Groups and Symmetry: A Guide to Discovering Mathematics,* AMS 1996.

* Stillwell, John, *Mathematics and its History,* Springer-Verlag, 2002.

* Tatersall, James J., *Elementary Number Theory,* Cambridge University Press, 1999.

* Taha, Hamdy A., *Operations Research: An Introduction,* 5th ed., Prentice Hall, 1995.

* Various issues of *Ganita Bharati,* the Bulletin of the Indian Socieity for History of Mathematics.

* Website: www.maths.iupui.edu/m290 (for Perspective and Art).

ELEMENTS OF ANALYSIS

Economics (II Year)

(5 lectures per week + 1 tutorial)

Unit I: Real Sequences 16 marks

Finite and infinite sets examples of countable and uncountable sets. Real line; absolute value bounded sets suprema and infima, statement of order Completeness property of R, Archimedean property of R, intervals. Real sequences, Convergence, sum and product of convergent sequences, proof of convergence of some simple sequences such as $(-1)^n/n, 1/n^2$,

$(1+1/n)^n$, x^n with $|x|<1$, a_n/n, where a_n is a bounded sequence. Concept of cluster points and statement of Bolzano Weierstrass' theorem. Statement and illustration of Cauchy convergence criterion for sequences. Cauchy's theorem on limits, order preservation and squeeze theorem, monotone sequences and their convergence.

Unit II: Infinite Series 12 marks

Definition and a necessary condition for convergence of an infinite series. Cauchy convergence criterion for series, positive term series, geometric series, comparison test, limit comparison test, convergence of p-series, Root test, Ratio test, alternating series, Leibnitz's test. Definition and examples of absolute and conditional convergemce.

Unit III: Power series 10 marks

Definition of power series: radius of convergence, Cauchy-Hadamard theorem, statement and illustration of term-by-term differentiation and integration of power series. Power series expansions for exp(x), sin(x), cos(x), log(1+x) and their properties.

Recommended books:

1. R.G. Bartle and D.R. Sherbert: Introduction to Real Analysis, John Wiley and Sons (Asia) Pte. Ltd., 2000.

2. C. P. Simon and L. Blume: Mathematics for Economists, W W Norton and Company,1994.

3. K. Sydsaeter and P.J. Hammod, Mathematics for Economics Analysis, Pearson Education, 2002.

ALGEBRA AND CALCULUS

(Other than Economics) (II year)

(5 lectures per week + 1 tutorial)

Unit-I Algebra and Geometry 12 marks

R, R^2, R^3 as Vector Spaces over R. Standard basis for each of them. Concept of linear independence. Matrices, basis concepts and algebraic operations. Determinants. System of linear equations and their solutions. Methods for finding inverse of matrix, basic concepts and algebraic operations. Determinants, System of linear equations and their solutions. Methods for finding inverse of a matrix.

Cramer's rule. Solution of problems arising in psychology, geography and other Social Sciences.

Cartesian Coordinates in 2 and 3 dimensions. Techniques of tracing of standard curves like straight lines, circle, ellipse, parabola and hyperbola. Reflection Property.

Interpretation of Equations for surfaces like Sphere, Cones, Ellipsoid and their pictures.

Functions and analysis of graphical information. Graphs of functions such as polynomials, trigonometric functions, exponential and logarithmic functions, inverse trigonometric functions arising in problems of geography, political science, psychology etc. such as growth, decay and population growth. Concept of shifting and scaling of graphs.

Unit-II: Calculus-I 12 marks

Concept of limit and continuity along with the intuitive and graphical approach. Input output examples for motivation.

Statement and implication of intermediate value theorem and other properties such as maximum and minimum attained for continuous functions on closed bounded intervals. Examples from common everyday phenomena related to social sciences and humanities. Differentiation and derivatives of first and second order. Statement and interpretation of algebra of derivatives. The derivative in graphing and applications. Slope of a graph, tangent lines. Increasing and decreasing behaviour of a function. Concavity and convexity. Maxima, minima and point of inflexion. Applications to problems in Social Sciences.

Statements, interpretation and practical applications of Rolle's theorem and Lagrange's mean value theorem.

Sequences to be introduced through the examples arising in social sciences beginning with finite sequences, followed by concepts of recursion and difference equations. The Fibonacci sequence arising from the branching habit of trees and the breeding habit of rabbits.

Intutive idea of the convergence of series, especially geometric series. Taylor and Maclaurin series formula for exp x. log (1+x), sin x, cos x.

Unit-III: Calculus-II 12 marks

Integration of simple trigonometric, rational and irrational functions. Concept of definite integral as sum of limits. Calculation of area and length of curves with emphasis on solution of problems arising in geography, psychology and other social sciences. Mathematical modeling with differential equations of first order. Formulation and verification of solution of differential equations such as population growth and spread of disease.

References:

1. H. Anton, I. Bivens and S. Davis: Calculus, John Wiley and Sons (Asia), 2002.

2. B. Thomas, R.L. Finey: Calculus and Analytic Geometry, Pearson Education (Singapore), 2001.

3. T.M.Apostol, Calculus, volume I. John Wiley and Sons (Asia) Ltd., 2002.

4. H.S. Bear, Understanding Calculus, John Wiley and Sons, 2003.